Esquisse d'un Programme

"Esquisse d'un Programme" is a famous proposal for long-term mathematical research made by the German-born, French mathematician Alexander Grothendieck [1] . He pursued the sequence of logically linked ideas in his important project proposal from 1984 until 1988, but his proposed research continues to date to be of major interest in several branches of advanced mathematics. Grothendieck's vision provides inspiration today for several developments in mathematics such as the extension and generalization of Galois theory, which is currently being extended based on his original proposal.

Outline of paper's content

Submitted in 1984, the *Esquisse d'un Programme*[2] was a successful proposal submitted by Alexander Grothendieck for a position at the Centre National de la Recherche Scientifique, which Grothendieck held from 1984 till 1988.[3] This proposal was however not formally published until 1997, because the author "could not be found, much less his permission requested".[4] The dessins d'enfants, or "children's drawings" and "anabelian algebraic geometry" — non-Abelian algebraic topology and noncommutative geometry — that are contained in this long-term program, continue even today to inspire extensive mathematical studies.

Abstract of the paper

("*Sommaire*")

- 1. The Proposal and enterpise ("Envoi").
- 2. "Teichmüller's Lego-game and the Galois group of Q over Q" ("Un jeu de “Lego-Teichmüller” et le groupe de Galois de Q sur Q").
- 3. Number fields associated with "dessin d'enfants". ("Corps de nombres associés à un dessin d’enfant").
- 4. Regular polyhedra over finite fields ("Polyèdres réguliers sur les corps finis").
- 5. General topology or a 'Moderated Topology' ("Haro sur la topologie dite 'générale', et réflexions heuristiques vers une topologie dite 'modérée").
- 6. Differentiable theories and moderated theories ("Théories différentiables" (à la Nash) et “théories modérées").
- 7. Pursuing Stacks ("À la Poursuite des Champs"}[5] .
- 8. Two-dimensional geometry ("Digressions de géométrie bidimensionnelle"[6] .
- 9. Summary of proposed studies ("Bilan d’une activité enseignante").
- 10. Epilogue.
- Notes

Suggested further reading for the interested mathematical reader is provided in the *References* section.

The Long March across the Theory of Galois

“This manuscript, consisting of some nearly 800 hand-written double pages, dating from 1981, was left behind with Grothendieck's other unpublished manuscripts when he disappeared in 1991. Typed in Tex, it comes out to about 400 pages. It goes together with a further 1,000 pages or so of additional notes and sections which have not yet been read or typed. Many of the major themes were summarised in the 1983 manuscript Esquisse d'un Programme.”

The Table of Contents for this important work by Alexander Grothendieck was originally compiled in French by the author and is reproduced here after the English Translation of the major parts of the Long March.

Table of Contents for the Long March across Galois Theory

Extensions of Galois's theory for groups: Galois groupoids, categories and functors

Galois has developed a powerful, fundamental algebraic theory in mathematics that provides very efficient computations for certain algebraic problems by utilizing the algebraic concept of groups, which is now known as the theory of Galois groups; such computations were not possible before, and also in many cases are much more effective than the 'direct' calculations without using groups[7] . To begin with, Alexander Grothendieck stated in his proposal: "*Thus, the group of Galois is realized as the automorphism group of a concrete, pro-finite group which respects certain structures that are essential to this group.*" This fundamental, Galois group theory in mathematics has been considerably expanded, at first to groupoids- as proposed in Alexander Grothendieck's *Esquisse d' un Programme* (*EdP*)- and now already partially carried out for groupoids; the latter are now further developed beyond groupoids to categories by several groups of mathematicians. Here, we shall focus only on the well-established and fully validated extensions of Galois' theory. Thus, EdP also proposed and anticipated, along previous Alexander Grothendieck's *IHÉS* seminars (SGA1 to SGA4) held in the 1960s, the development of even more powerful extensions of the original Galois's theory for groups by utilizing categories, functors and natural transformations, as well as further expansion of the manifold of ideas presented in Alexander Grothendieck's *Descent Theory*. The notion of motive has also been pursued actively [8] . This was developed into the motivic Galois group, Grothendieck topology and Grothendieck category [9] . Such developments were recently extended in algebraic topology *via* representable functors [10] and the fundamental groupoid functor [11].

See also

- Alexander Grothendieck
- Grothendieck's Galois theory
- Grothendieck group
- Grothendieck category [12]
- Grothendieck inequality or Grothendieck constant
- Grothendieck–Katz p-curvature conjecture
- Grothendieck's relative point of view
- Grothendieck–Riemann–Roch theorem
- Grothendieck's Séminaire de géométrie algébrique
- Grothendieck space
- Grothendieck spectral sequence
- Grothendieck topology
- Grothendieck universe
- Tarski–Grothendieck set theory
- IHES
- IHES at Forty by Allyn Jackson [13]

Notes

[1] Scharlau, Winifred (September 2008), written at Oberwolfach, Germany, "Who is Alexander Grothendieck", *Notices of the American Mathematical Society* (Providence, RI: American Mathematical Society) 55(8): 930–941, ISSN 1088-9477, OCLC 34550461, http://www.ams.org/notices/200808/tx080800930p.pdf

[2] Alexander Grothendieck, 1984. "Esquisse d'un Programme", (1984 manuscript), finally published in Schneps and Lochak (1997, I), pp.5-48; English transl., ibid., pp. 243-283. MR 99c:14034

[3] Rehmeyer, Julie (May 9, 2008), "Sensitivity to the Harmony of Things", *Science News*

[4] Schneps and Lochak (1997, I) p.1

[5] http://www.bangor.ac.uk/r.brown/pstacks.htm

[6] Cartier, Pierre (2001), "A mad day's work: from Grothendieck to Connes and Kontsevich The evolution of concepts of space and symmetry", *Bull. Amer. Math. Soc.* **38**(4): 389–408, <http://www.ams.org/bull/2001-38-04/S0273-0979-01-00913-2/S0273-0979-01-00913-2.pdf>. An English translation of Cartier (1998)

[7] Cartier, Pierre (1998), "La Folle Journée, de Grothendieck à Connes et Kontsevich — Évolution des Notions d'Espace et de Symétrie", *Les Relations entre les Mathématiques et la Physique Théorique — Festschrift for the 40th anniversary of the* IHÉS*, Institut des Hautes Études Scientifiques*, pp. 11–19

[8] http://en.wikipedia.org/wiki/Motive_(algebraic_geometry)

[9] http://planetmath.org/encyclopedia/GrothendieckCategory.html

[10] http://planetphysics.org/encyclopedia/RepresentableFunctor.html

[11] http://planetphysics.org/encyclopedia/QuantumFundamentalGroupoid3.html

[12] http://planetphysics.org/encyclopedia/GrothendieckCategory.html

[13] http://www.ams.org/notices/199903/ihes-changes.pdf

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- Fundamental Groupoid Functor (http://planetphysics.org/encyclopedia/QuantumFundamentalGroupoid3.html)

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