LUKASIEWICZ-MOISIL MANY-VALUED LOGIC ALGEBRA OF HIGHLY-COMPLEX SYSTEMS

Ion C. Baianu, George Georgescu,
James F. Glazebrook and Ronald Brown

Abstract.
The fundamentals of Lukasiewicz-Moisil logic algebras and their applications to complex genetic network dynamics and highly complex systems will be presented in the context of a categorical ontology theory of levels, Medical Bioinformatics and Self-organizing, Highly Complex Systems.

Keywords: *LM-logic algebra, algebraic category of LM-logic algebras, fundamental theorems of LM-logic algebra, many-valued logics of highly complex systems and Categorical Ontology*

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1. Algebraic Logic, Operational and Lukasiewicz Quantum Logic

As pointed out by Birkhoff and von Neumann (1936), a logical foundation of quantum mechanics consistent with quantum algebra is essential for both the completeness and mathematical validity of the theory. The development of Quantum Mechanics from its very beginnings both inspired and required the consideration of specialized logics compatible with a new theory of measurements for microphysical systems. Such a specialized logic was initially formulated by Birkhoff and von Neumann (1936) and called 'Quantum Logic'. Subsequent research on Quantum Logics (Dalla Chiara, 2004) resulted in several approaches that involve several types of non-distributive lattice (algebra) for n–valued quantum logics. Thus, modifications of the Łukasiewicz Logic Algebras that were introduced in the context of algebraic categories by Georgescu and Vraciu (1973), also recently reviewed and expanded by Georgescu (2006), can provide an appropriate framework for representing quantum systems, or– in their unmodified form- for describing the activities of complex networks in categories of Łukasiewicz Logic Algebras (Baianu, 1977).

There is however a serious problem of logical inconsistency between the quantum algebra and the Heyting logic algebra as a candidate for quantum logic (Baianu et al 2007b). Furthermore, quantum algebra and topological approaches that are ultimately based on set-theoretical concepts and differentiable spaces (manifolds) also encounter serious problems of internal inconsistency. Since it has been shown that standard set theory which is subject to the axiom of choice relies on Boolean logic (Mac Lane and Moerdijk, 2000), there appears to exist a basic logical inconsistency between the quantum logic–which is not Boolean–and the Boolean logic underlying all differentiable manifold approaches that rely on continuous spaces of points, or certain specialized sets of elements. A possible solution to such inconsistencies is the definition of a generalized Topos concept, and more specifically, of a Quantum, Extended Topos concept which is consistent with both Quantum Logic and Quantum Algebras, being thus suitable as a framework for unifying quantum field theories and physical modelling of complex systems and systems biology.

2. Lattices and Von Neumann-Birkhoff (VNB) Quantum Logic: Definition and Some Logical Properties

We commence here by giving the *set-based Definition of a Lattice*. An *s–lattice* **L**, or a 'set-based' lattice, is defined as a *partially ordered set* that has all binary products (defined by the *s*–lattice operation " $\bigwedge$") and coproducts (defined by the *s*–lattice operation "$\bigvee$ "), with the "partial ordering" between two elements X and Y belonging to the *s*–lattice being written as "$X \preceq Y$". The partial order defined by $\preceq$ holds in **L** as $X \preceq Y$ if and only if $X = X \bigwedge Y$ (or equivalently, $Y = X \bigvee Y$ Eq.(3.1)(p. 49 of Mac Lane and Moerdijk, 1992). A *lattice* can also be defined as a *category* (see, for example: Lawvere, 1966; Baianu, 1970; Baianu et al., 2004b) subject to all ETAC axioms, (but not subject, in general, to the Axiom of Choice usually encountered with sets relying on (distributive) Boolean Logic), that has all binary products and all binary coproducts, as well as the following 'partial ordering' properties:

Łukasiewicz-Moisil (LM) Quantum Logic (LQL) and Algebras.

With all truth 'nuances' or assertions of the type $<<$ *system A* is excitable to the *i*-th level and system B is excitable to the *j*-th level $>>$ one can define a special type of lattice that subject to the axioms introduced by Georgescu and Vraciu (1970) becomes a *n-valued Łukasiewicz-Moisil, or LM, Algebra*. Further algebraic and logic details are provided in Georgescu (2006) and Baianu et al (2007b).

In order to have the n-valued Łukasiewicz Logic Algebra represent correctly the basic behaviour of quantum systems (observed through measurements that involve a quantum system interactions with a measuring instrument –which is a macroscopic object, several of these axioms have to be significantly changed so that the resulting lattice becomes non-distributive and also (possibly) non–associative (Dalla Chiara, 2004). On the other hand, for classical systems, modelling with the unmodified Łukasiewicz Logic Algebra can also include both stochastic and fuzzy behaviours. For an example of such models the reader is referred to a previous publication (Baianu, 1977) modelling the activities of complex genetic networks from a classical standpoint. One can also define as in (Georgescu and Vraciu, 1970) the 'centers' of certain types of Łukasiewicz n-Logic Algebras; then one has the following important theorem for such Centered Łukasiewicz n-Logic Algebras which actually defines an equivalence relation.

The Adjointness Theorem: (Georgescu and Vraciu, 1970).

There exists an Adjointness between the Category of Centered Łukasiewicz n-Logic Algebras, **CLuk**–n, and the Category of Boolean Logic Algebras (**Bl**).

Remarks (1) The natural equivalence logic classes defined by the adjointness relationships in the above Adjointness Theorem define a fundamental, *'logical groupoid'* structure.

(2) In order to adapt the standard Łukasiewicz Logic Algebra to the appropriate Quantum Łukasiewicz Logic Algebra, LQL, a few axioms of LM-algebra need modifications, such as : $N(N(X)) = Y \neq X$ (instead of the restrictive identity $N(N(X)) = X$, whenever the context, 'reference frame for the measurements', or 'measurement preparation' interaction conditions for quantum systems are incompatible with the standard 'negation' operation N of the Łukasiewicz Logic Algebra; the latter remains however valid for classical or semi–classical systems, such as various complex networks with n-states (cf. Baianu, 1977). Further algebraic and conceptual details are provided in a rigorous review by Georgescu (2006), and also in the recently published reports by Baianu et al (2007b) and Brown et al. (2007).

3.The third section

Include as many sections as necessary

4.Conclusions

References

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I. C. Baianu, PhD., Professor, ACES College and NPRE Department, University of Illinois at Urbana-Champaign, Urbana IL 61801, USA
email:*ibaianu@illinois.edu*