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ALGEBRAIC TOPOLOGY FOUNDATIONS OF SUPERSYMMETRY AND SYMMETRY BREAKING IN QUANTUM FIELD THEORY AND QUANTUM GRAVITY.

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Abstract. A novel Algebraic Topology approach to Supersymmetry (SUSY) and Symmetry Breaking in Quantum Field and Quantum Gravity theories is presented with a view to developing a wide range of physical applications (such as, controlled nuclear fusion and other nuclear reactions studies in quantum chromodynamics, nonlinear physics at high energy densities, dynamic Jahn-Teller effects, superfluidity, high temperature superconductors, multiple scattering by molecular systems, molecular or atomic paracrystal structures, nanomaterials, ferromagnetism in glassy materials, spin glasses, quantum phase transitions, supergravity, and so on). This approach requires a unified conceptual framework that utilizes extended symmetries and quantum groupoid, algebroid and functorial representations of non–Abelian higher dimensional structures pertinent to quantized spacetime topology and state space geometry of quantum operator algebras. The relevance of our approach to extended quantum symmetries and their associated representations in locally covariant General Relativity theories that are consistent with nonlocal quantum field theories will also be discussed.

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addl. Thanks ...

We will explain these difficulties by looking first at the case of a finite group where this whole procedure presents no problems at all. If G is a finite group, the C*-algebra $C_0(G)$ is the *-algebra $C(G)$ of all complex functions on G with pointwise operations. It is of course finite-dimensional so that it suffices to look at the algebraic tensor product $C(G) \odot C(G)$. The *multiplier algebra* is also defined by this algebraic tensor product. In particular, the *comultiplication* Φ is a comultiplication in the ordinary, algebraic sense on the algebra $C(G)$. The properties

$$ep = pe = p$$

and

$$p^{-1}p = pp^{-1} = e$$

for all p, become in terms of ϵ and κ as follows:

$$(\epsilon \otimes \iota)\Phi(f) = (\iota \otimes \epsilon)\Phi(f) = f$$

and

$$m(\kappa \otimes \iota)\Phi(f) = m(\iota \otimes \kappa)\Phi(f) = f$$

$\sum$

where ι denotes the identity map , and where m is the multiplication defined as a map from the tensor product $C(G) \odot C(G)$ to $C(G)$ defined by $m(f \otimes g) = fg$. This means that $(C(G), \Phi)$ is a Hopf algebra. In fact, it is a Hopf *-algebra if we let

$$f^*(p) = \overline{f(p)}$$

. Let us recall the definition of a Hopf *-algebra (see e.g. [22]) :

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