Metatheories

Metalogic, Algebraic Logic and Metamathematics

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Metatheories

Metatheory

A **metatheory** or **meta-theory** is a theory whose subject matter is some other theory. In other words it is a theory about a theory. Statements made in the metatheory about the theory are called metatheorems.

According to the systemic TOGA meta-theory [1] , a meta-theory may refer to the specific point of view on a theory and to its subjective meta-properties, but not to its application domain. In the above sense, a theory **T** of the domain **D** is a meta-theory if **D** is a theory or a set of theories. A general theory is not a meta-theory because its domain **D** are not theories.

The following is an example of a meta-theoretical statement:[2]

> “ Any physical theory is always provisional, in the sense that it is only a hypothesis; you can never prove it. No matter how many times the results of experiments agree with some theory, you can never be sure that the next time the result will not contradict the theory. On the other hand, you can disprove a theory by finding even a single observation that disagrees with the predictions of the theory. ”

Meta-theory belongs to the philosophical specialty of epistemology and → metamathematics, as well as being an object of concern to the area in which the individual theory is conceived. An emerging domain of meta-theories is systemics.

Taxonomy

Examining groups of related theories, a first finding may be to identify classes of theories, thus specifying a taxonomy of theories. A proof engendered by a metatheory is called a *metatheorem*.

History

The concept burst upon the scene of twentieth-century philosophy as a result of the work of the German mathematician David Hilbert, who in 1905 published a proposal for proof of the consistency of mathematics, creating the field of → metamathematics. His hopes for the success of this proof were dashed by the work of Kurt Gödel who in 1931 proved this to be unattainable by his incompleteness theorems. Nevertheless, his program of unsolved mathematical problems, out of which grew this metamathematical proposal, continued to influence the direction of mathematics for the rest of the twentieth century.

The study of metatheory became widespread during the rest of that century by its application in other fields, notably scientific linguistics and its concept of metalanguage.

References

[1] * Meta-Knowledge Unified Framework (http://hid.casaccia.enea.it/Meta-know-1.htm) - the TOGA meta-theory
[2] Stephen Hawking in *A Brief History of Time*

See also

- meta-
- meta-knowledge
- → Metalogic
- → Metamathematics

External links

- Meta-theoretical Issues (2003), Lyle Flint (http://www.bsu.edu/classes/flint/comm360/metatheo.html)

Metalogics

Metalogic

Metalogic is the study of the → metatheory of logic. While *logic* is the study of the manner in which logical systems can be used to decide the correctness of arguments, metalogic studies the properties of the logical systems themselves.[1] According to Geoffrey Hunter, while logic concerns itself with the "truths of logic," metalogic concerns itself with the theory of "sentences used to express truths of logic."[2]

The basic objects of study in metalogic are formal languages, formal systems, and their interpretations. The study of interpretation of formal systems is the branch of mathematical logic known as model theory, while the study of deductive apparatus is the branch known as proof theory.

History

Metalogical questions have been asked since the time of Aristotle. However, it was only with the rise of formal languages in the late 19th and early 20th century that investigations into the foundations of logic began to flourish. In 1904, David Hilbert observed that in investigating the foundations of mathematics that logical notions are presupposed, and therefore a simultaneous account of metalogical and → metamathematical principles was required. Today, metalogic and metamathematics are largely synonymous with each other, and both have been substantially subsumed by mathematical logic in academia.

Important distinctions in metalogic

Metalanguage-Object language

In metalogic, formal languages are sometimes called *object languages*. The language used to make statements about an object language is called a *metalanguage*. This distinction is a key difference between logic and metalogic. While logic deals with *proofs in a formal system*, expressed in some formal language, metalogic deals with *proofs about a formal system* which are expressed in a metalanguage about some object language.

Syntax-semantics

In metalogic, 'syntax' has to do with formal languages or formal systems without regard to any interpretation of them, whereas, 'semantics' has to do with interpretations of formal languages. The term 'syntactic' has a slightly wider scope than 'proof-theoretic', since it may be applied to properties of formal languages without any deductive systems, as well as to formal systems. 'Semantic' is synonymous with 'model-theoretic'.

Use-mention

In metalogic, the words 'use' and 'mention', in both their noun and verb forms, take on a technical sense in order to identify an important distinction.[2] The *use–mention distinction* (sometimes referred to as the *words-as-words distinction*) is the distinction between *using* a word (or phrase) and *mentioning* it. Usually it is indicated that an expression is being mentioned rather than used by enclosing it in quotation marks, printing it in italics, or setting the expression by itself on a line. The enclosing in quotes of an expression gives us the name of an expression, for example:

> 'Metalogic' is the name of this article.
>
> This article is about metalogic.

Type-token

The *type-token distinction* is a distinction in metalogic, that separates an abstract concept from the objects which are particular instances of the concept. For example, the particular bicycle in your garage is a token of the type of thing known as "The bicycle." Whereas, the bicycle in your garage is in a particular place at a particular time, that is not true of "the bicycle" as used in the sentence: "*The bicycle* has become more popular recently." This distinction is used to clarify the meaning of symbols of formal languages.

Overview

Formal language

A *formal language* is an organized set of symbols the essential feature of which is that it can be precisely defined in terms of just the shapes and locations of those symbols. Such a language can be defined, then, without any reference to any meanings of any of its expressions; it can exist before any interpretation is assigned to it -- that is, before it has any meaning. First order logic is expressed in some formal language. A formal grammar determines which symbols and sets of symbols are formulas in a formal language.

A formal language can be defined formally as a set *A* of strings (finite sequences) on a fixed alphabet α. Some authors, including Carnap, define the language as the ordered pair <α, *A*>.[3] Carnap also requires that each element of α must occur in at least one string in *A*.

Formal grammar

A *formal grammar* (also called *formation rules*) is a precise description of a the well-formed formulas of a formal language. It is synonymous with the set of strings over the alphabet of the formal language which constitute well formed formulas. However, it does not describe their semantics (i.e. what they mean).

Formal systems

A *formal system* (also called a *logical calculus*, or a *logical system*) consists of a formal language together with a deductive apparatus (also called a *deductive system*). The deductive apparatus may consist of a set of transformation rules (also called *inference rules*) or a set of axioms, or have both. A formal system is used to derive one expression from one or more other expressions.

A *formal system* can be formally defined as an ordered triple $<\alpha, \mathcal{I}, \mathcal{D}d>$, where $\mathcal{D}d$ is the relation of direct derivability. This relation is understood in a comprehensive sense such that the primitive sentences of the formal system are taken as directly derivable from the empty set of sentences. Direct derivability is a relation between a sentence and a finite, possibly empty set of sentences. Axioms are laid down in such a way that every first place member of $\mathcal{D}d$ is a member of $\mathcal{I}$ and every second place member is a finite subset of $\mathcal{I}$.

It is also possible to define a *formal system* using only the relation $\mathcal{D}d$. In this way we can omit $\mathcal{I}$, and α in the definitions of *interpreted formal language*, and *interpreted formal system*. However, this method can be more difficult to understand and work with.[3]

Formal proofs

A *formal proof* is a sequences of well-formed formulas of a formal language, the last one of which is a theorem of a formal system. The theorem is a syntactic consequence of all the well formed formulae preceding it in the proof. For a well formed formula to qualify as part of a proof, it must be the result of applying a rule of the deductive apparatus of some formal system to the previous well formed formulae in the proof sequence.

Interpretations

An *interpretation* of a formal system is the assignment of meanings, to the symbols, and truth-values to the sentences of the formal system. The study of interpretations is called Formal semantics. *Giving an interpretation* is synonymous with *constructing a model.*

Results in metalogic

Results in metalogic consist of such things as formal proofs demonstrating the consistency, completeness, and decidability of particular formal systems.

Major results in metalogic include:

- Proof of the uncountability of the set of all subsets of the set of natural numbers (Cantor's theorem 1891)
- Löwenheim-Skolem theorem (Leopold Löwenheim 1915 and Thoralf Skolem 1919)
- Proof of the consistency of truth-functional propositional logic (Emil Post 1920)
- Proof of the semantic completeness of truth-functional propositional logic (Paul Bernays 1918)[4] ,(Emil Post 1920)[2]
- Proof of the syntactic completeness of truth-functional propositional logic (Emil Post 1920)[2]
- Proof of the decidability of truth-functional propositional logic (Emil Post 1920)[2]
- Proof of the consistency of first order monadic predicate logic (Leopold Löwenheim 1915)
- Proof of the semantic completeness of first order monadic predicate logic (Leopold Löwenheim 1915)
- Proof of the decidability of first order monadic predicate logic (Leopold Löwenheim 1915)
- Proof of the semantic completeness of first order predicate logic (Gödel's completeness theorem 1930)
- Proof of the consistency of first order predicate logic (David Hilbert and Wilhelm Ackermann 1928)
- Proof of the semantic completeness of first order predicate logic (Kurt Gödel 1930)
- Proof of the undecidability of first order predicate logic (Alonzo Church 1936)
- Gödel's first incompleteness theorem 1931

- Gödel's second incompleteness theorem 1931

See also

- → Metamathematics
- Formal semantics

References

[1] Harry J. Gensler, Introduction to Logic, Routledge, 2001, p. 253.
[2] Hunter, Geoffrey, Metalogic: An Introduction to the Metatheory of Standard First-Order Logic, University of California Pres, 1971
[3] Rudolf Carnap (1958) *Introduction to Symbolic Logic and its Applications*, p. 102.
[4] Hao Wang, Reflections on Kurt Gödel

Algebraic Logic

Algebraic logic

In mathematical logic, **algebraic logic** formalizes logic using the methods of abstract algebra.

Algebras as models of logics

Algebraic logic treats algebraic structures, often bounded lattices, as models (interpretations) of certain logics, making logic a branch of order theory.

In algebraic logic:

- Variables are tacitly universally quantified over some universe of discourse. There are no existentially quantified variables or open formulas;
- Terms are built up from variables using primitive and defined operations. There are no connectives;
- Formulas, built from terms in the usual way, can be equated if they are logically equivalent. To express a tautology, equate a formula with a truth value;
- The rules of proof are the substitution of equals for equals, and uniform replacement. Modus ponens remains valid, but is seldom employed.

In the table below, the left column contains one or more logical or mathematical systems, and the algebraic structure which are its models are shown on the right in the same row. Some of these structures are either Boolean algebras or proper extensions thereof. Modal and other nonclassical logics are typically modeled by what are called "Boolean algebras with operators."

Algebraic formalisms going beyond first-order logic in at least some respects include:

- Combinatory logic, having the expressive power of set theory;
- Relation algebra, arguably the paradigmatic algebraic logic, can express Peano arithmetic and most axiomatic set theories, including the canonical ZFC.

logical system	**its models**
Classical sentential logic	Lindenbaum-Tarski algebra Two-element Boolean algebra
Intuitionistic propositional logic	Heyting algebra
Łukasiewicz logic	MV-algebra
Modal logic K	Modal algebra
Lewis's S4	Interior algebra
Lewis's S5; Monadic predicate logic	Monadic Boolean algebra
First-order logic	Cylindric algebra Polyadic algebra Predicate functor logic
Set theory	Combinatory logic Relation algebra

History

On the history of algebraic logic before World War II, see Brady (2000) and Grattan-Guinness (2000) and their ample references. On the postwar history, see Maddux (1991) and Quine (1976).

Algebraic logic has at least two meanings:

- The study of Boolean algebra, begun by George Boole, and of relation algebra, begun by Augustus DeMorgan, extended by Charles Sanders Peirce, and taking definitive form in the work of Ernst Schröder;
- Abstract algebraic logic, a branch of contemporary mathematical logic.

Perhaps surprisingly, algebraic logic is the oldest approach to formal logic, arguably beginning with a number of memoranda Leibniz wrote in the 1680s, some of which were published in the 19th century and translated into English by Clarence Lewis in 1918. But nearly all of Leibniz's known work on algebraic logic was published only in 1903, after Louis Couturat discovered it in Leibniz's Nachlass. Parkinson (1966) and Loemker (1969) translated selections from Couturat's volume into English.

Brady (2000) discusses the rich historical connections between algebraic logic and model theory. The founders of model theory, Ernst Schroder and Leopold Loewenheim, were logicians in the algebraic tradition. Alfred Tarski, the founder of set theoretic model theory as a major branch of contemporary mathematical logic, also:

- Co-discovered Lindenbaum-Tarski algebra;
- Invented cylindric algebra;
- Wrote the 1940 paper that revived relation algebra, and that can be seen as the starting point of abstract algebraic logic.

Modern mathematical logic began in 1847, with two pamphlets whose respective authors were Augustus DeMorgan and George Boole. They, and later C.S. Peirce, Hugh MacColl, Frege, Peano, Bertrand Russell, and A. N. Whitehead all shared Leibniz's dream of combining symbolic logic, mathematics, and philosophy. Relation algebra is arguably the culmination of Leibniz's approach to logic. With the exception of some writings by Leopold Loewenheim and Thoralf Skolem, algebraic logic went into eclipse soon after the 1910-13 publication of *Principia Mathematica*, not to revive until Tarski's 1940 reexposition of relation algebra.

Leibniz had no influence on the rise of algebraic logic because his logical writings were little studied before the Parkinson and Loemker translations. Our present understanding of Leibniz the logician stems mainly from the work of Wolfgang Lenzen, summarized in Lenzen (2004). [1] To see how present-day work in logic and metaphysics can draw inspiration from, and shed light on, Leibniz's thought, see Zalta (2000). [2]

See also

- Abstract algebraic logic
- Algebraic structure
- Boolean algebra (logic)
- Boolean algebra (structure)
- Cylindric algebra
- Lindenbaum-Tarski algebra
- Mathematical logic
- Model theory
- Monadic Boolean algebra
- Predicate functor logic
- Relation algebra
- Universal algebra

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External links

- Stanford Encyclopedia of Philosophy: "Propositional Consequence Relations and Algebraic Logic [6]" -- by Ramon Jansana.

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[1] http://www.philosophie.uni-osnabrueck.de/Publikationen%20Lenzen/Lenzen%20Leibniz%20Logic.pdf
[2] http://mally.stanford.edu/Papers/leibniz.pdf
[3] http://www.elsevier.com/wps/find/bookdescription.cws_home/621535/description
[4] http://www.philosophie.uni-osnabrueck.de/Publikationen%20Lenzen/Lenzen%20Leibniz%20Logic.pdf
[5] http://mally.stanford.edu/leibniz.pdf
[6] http://plato.stanford.edu/entries/consequence-algebraic/

Many-Valued Logics

Multi-valued logic

Multi-valued logics are 'logical calculi' in which there are more than two truth values. Traditionally, in 'logical calculi' - invented by Aristotle - there were only two possible values (i.e. TRUE and FALSE) for any proposition to take. An obvious extension to classical two-valued logic is an n-valued logic for $n > 2$. Those most popular in the literature are three-valued (e.g. Łukasiewicz's and Kleene's) which accept the values TRUE, FALSE, UNKNOWN, the finite-valued with more than 3 values, and infinite-valued (e.g. → fuzzy logic) ones.

Relation to classical logic

Logics are usually systems intended to codify rules for preserving some semantic property of propositions across transformations. In classical logic, this property is "truth." In a valid argument, the truth of the derived proposition is guaranteed if the premises are jointly true, because the application of valid steps preserves the property. However, that property doesn't have to be that of "truth"; instead, it can be some other concept.

Multi-valued logics are intended to preserve the property of designationhood (or being designated). Since there are more than two truth values, rules of inference may be intended to preserve more than just whichever corresponds (in the relevant sense) to truth. For example, in a three-valued logic, sometimes the two greatest truth-values (when they are represented as e.g. positive integers) are designated and the rules of inference preserve these values. Precisely, a valid argument will be such that the value of the premises taken jointly will always be less than or equal to the conclusion.

For example, the preserved property could be *justification*, the foundational concept of intuitionistic logic. Thus, a proposition is not true or false; instead, it is justified or flawed. A key difference between justification and truth, in this case, is that the law of the excluded middle doesn't hold: a proposition that is not flawed is not necessarily justified; instead, it's only not proven that it's flawed. The key difference is the determinacy of the preserved property: One may prove that P is justified, that P is flawed, or be unable to prove either. A valid argument preserves justification across transformations, so a proposition derived from justified propositions is still justified. However, there are proofs in classical logic that depend upon the law of excluded middle; since that law is not usable under this scheme, there are propositions that cannot be proven that way.

Relation to fuzzy logic

Multi-valued logic is strictly related with Fuzzy set theory and → fuzzy logic. The notion of fuzzy subset was introduced by Lotfi Zadeh as a formalization of vagueness; i.e., the phenomenon that a predicate may apply to an object not absolutely, but to a certain degree, and that there may be borderline cases. Indeed, as in multi-valued logic, fuzzy logic admits truth values different from "true" and "false". As an example, usually the set of possible truth values is the whole interval [0,1]. Nevertheless, the main difference between fuzzy logic and multi-valued logic is in the aims. In fact, in spite of its philosophical interest (it can be used to deal with the sorites paradox), fuzzy logic is devoted mainly to the applications. More precisely, there are two approaches to → Fuzzy logic. The first one is very closely linked with multi-valued logic tradition (Hajek school). So a set of designed values is fixed and this enables us to define an entailment relation. The deduction apparatus is defined by a suitable set of logical axioms and suitable inference rules. Another approach (Goguen, Pavelka and others) is devoted to defining a deduction apparatus in which *approximate reasonings* are admitted. Such an apparatus is defined by a suitable fuzzy subset of logical axioms and by a suitable set of fuzzy inference rules. In the first case the logical consequence operator gives the set of logical consequence of a given set of axioms. In the latter the logical consequence operator gives the fuzzy subset of logical consequence of a given fuzzy subset of hypotheses.

Another example of an infinitely-valued logic is probability logic.

History

The first known classical logician who didn't fully accept the law of the excluded middle was Aristotle (who, ironically, is also generally considered to be the first classical logician and the "father of logic"[1]), who admitted that his laws did not all apply to future events (*De Interpretatione, ch. IX*). But he didn't create a system of multi-valued logic to explain this isolated remark. The later logicians until the coming of the 20th century followed Aristotelian logic, which includes or implies the law of the excluded middle.

The 20th century brought the idea of multi-valued logic back. The Polish logician and philosopher Jan Łukasiewicz began to create systems of many-valued logic in 1920, using a third value "possible" to deal with Aristotle's paradox of the sea battle. Meanwhile, the American mathematician Emil L. Post (1921) also introduced the formulation of additional truth degrees with n>=2,where n are the truth values. Later Jan Łukasiewicz and Alfred Tarski together formulated a logic on n truth values where n>=2 and in 1932 Hans Reichenbach formulated a logic of many truth values where n→infinity. Kurt Gödel in 1932 showed that intuitionistic logic is not a finitely-many valued logic, and defined a system of Gödel logics intermediate between classical and intuitionistic logic; such logics are known as intermediate logics.

See also

- Fuzzy Logic
- Degrees of truth
- False dilemma
- Logical value
- MV-algebra
- IEEE 1164
- Perspectivism
- Rhizome (philosophy)
- Anekantavada
- Principle of Bivalence

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External links

- Stanford Encyclopedia of Philosophy: "Many-Valued Logic [2]" -- by Siegfried Gottwald.

Notes

[1] Hurley, Patrick. *A Concise Introduction to Logic*, 9th edition. (2006).
[2] http://plato.stanford.edu/entries/logic-manyvalued/

Metamathematics

Metamathematics

Metamathematics is the study of mathematics itself using mathematical methods. This study produces → metatheories, which are mathematical theories about other mathematical theories. Metamathematical metatheorems about mathematics itself were originally differentiated from ordinary mathematical theorems in the 19th century, to focus on what was then called the foundational crisis of mathematics. Richard's paradox (Richard 1905) concerning certain 'definitions' of real numbers in the English language is an example of the sort of contradictions which can easily occur if one fails to distinguish between mathematics and metamathematics.

The term "metamathematics" is sometimes used as a synonym for certain elementary parts of formal logic, including propositional logic and predicate logic.

History

Metamathematics was intimately connected to mathematical logic, so that the early histories of the two fields, during the late 19th and early 20th centuries, largely overlap. More recently, mathematical logic has often included the study of new pure mathematics, such as set theory, recursion theory, and pure model theory, which is not directly related to metamathematics.

Serious metamathematical reflection began with the work of Gottlob Frege, especially his *Begriffsschrift*.

David Hilbert was the first to invoke the term "metamathematics" with regularity (see Hilbert's program). In his hands, it meant something akin to contemporary proof theory, in which finitary methods are used to study various axiomatized mathematical theorems.

Other prominent figures in the field include Bertrand Russell, Thoralf Skolem, Emil Post, Alonzo Church, Stephen Kleene, Willard Quine, Paul Benacerraf, Hilary Putnam, Gregory Chaitin, and most important, Alfred Tarski and Kurt Gödel. In particular, Gödel's proof that, given any finite number of axioms for Peano arithmetic, there will be true statements about that arithmetic that cannot be proved from those axioms, a result known as the incompleteness theorem, is arguably the greatest achievement of metamathematics and the philosophy of mathematics to date.

Milestones

- Principia Mathematica (Whitehead and Russell 1925)
- Gödel's completeness theorem, 1930
- Gödel's incompleteness theorem, 1931
- Tarski's definition of model-theoretic satisfaction, now called the T-schema
- The proof of the impossibility of the Entscheidungsproblem, obtained independently in 1936–1937 by Church and Turing.

See also

- meta-
- model theory
- proof theory

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Categories

Fuzzy logics

Fuzzy logic

Fuzzy logic is a form of → multi-valued logic derived from fuzzy set theory to deal with reasoning that is approximate rather than precise. In contrast with binary sets having *binary logic*, also known as *crisp logic*, the fuzzy logic variables may have a membership value of only 0 or 1. Just as in fuzzy set theory with fuzzy logic the set membership values can range (inclusively) between 0 and 1, in fuzzy logic the degree of truth of a statement can range between 0 and 1 and is not constrained to the two truth values {true (1), false (0)} as in classic propositional logic.[1] And when *linguistic variables* are used, these degrees may be managed by specific functions, as discussed below.

The term "fuzzy logic" emerged as a consequence of the development of the theory of fuzzy sets by Lotfi Zadeh[2] .

In 1965 Lotfi Zadeh proposed fuzzy set theory[3] , and later established fuzzy logic based on fuzzy sets. Fuzzy logic has been applied to diverse fields, from control theory to artificial intelligence, yet still remains controversial among most statisticians, who prefer Bayesian logic, and some control engineers, who prefer traditional two-valued logic.

Ealier than Zadeh, a paper introducing the concept without using the term "fuzzy" was published by R.H. Wilkinson in 1963[4] and thus preceded fuzzy set theory. Wilkinson was the first one to redefine and generalize the earlier multivalued logics in terms of set theory. The main purpose of his paper, following his first proposals in his 1961 Electrical Engineering master thesis, was to show how any mathematical function could be simulated using hardwired analog electronic circuits. He did this by first creating various linear voltage ramps which were then selected in a "logic block" using diodes and resistor circuits which implemented the maximum and minimum Fuzzy Logic rules of the INCLUSIVE OR and the AND operations respectively. He called his logic "analog logic". Some say that the idea of fuzzy logic is set-theoretical equivalent of the "analog logic" of Wilkinson (without recourse to electrical circuits), but he never received any credit.

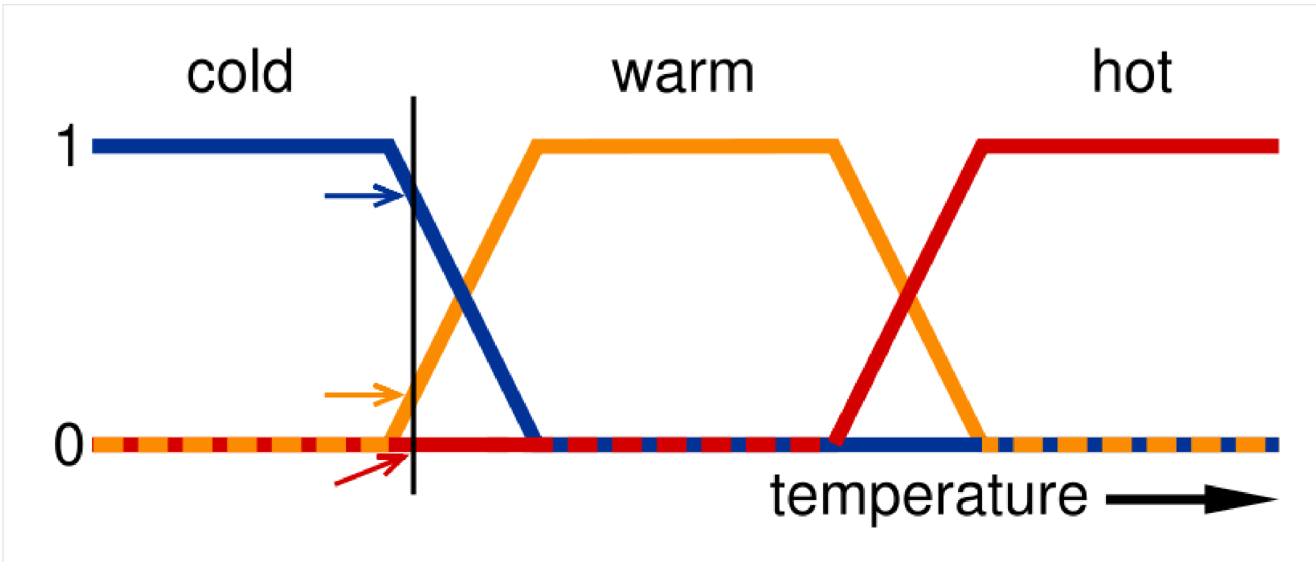
Degrees of truth

Both degrees of truth and probabilities range between 0 and 1 and hence may seem similar at first. However, they are distinct conceptually; truth represents membership in vaguely defined sets, not *likelihood* of some event or condition as in probability theory. For example, let a 100-ml glass contain 30 ml of water. Then we may consider two concepts: Empty and Full. The meaning of each of them can be represented by a certain fuzzy set. Then one might define the glass as being 0.7 empty and 0.3 full. Note that the concept of emptiness would be subjective and thus would depend on the observer or designer. Another designer might equally well design a set membership function where the glass would be considered full for all values down to 50 ml. It is essential to realize that fuzzy logic uses truth degrees as a mathematical model of the vagueness phenomenon while probability is a mathematical model of randomness.

A probabilistic setting would first define a scalar variable for the fullness of the glass, and second, conditional distributions describing the probability that someone would call the glass full given a specific fullness level. This model, however, has no sense without accepting occurrence of some event, e.g. that after a few minutes, the glass will be half empty. Note that the conditioning can be achieved by having a specific observer that randomly selects the label for the glass, a distribution over deterministic observers, or both. Consequently, probability has nothing in common with fuzziness, these are simply different concepts which superficially seem similar because of using the same interval of real numbers [0, 1]. Still, since theorems such as De Morgan's have dual applicability and properties of random variables are analogous to properties of binary logic states, one can see where the confusion might arise.

Applying truth values

A basic application might characterize subranges of a continuous variable. For instance, a temperature measurement for anti-lock brakes might have several separate membership functions defining particular temperature ranges needed to control the brakes properly. Each function maps the same temperature value to a truth value in the 0 to 1 range. These truth values can then be used to determine how the brakes should be controlled.



In this image, the meaning of the expressions *cold*, *warm*, and *hot* is represented by functions mapping a temperature scale. A point on that scale has three "truth values" — one for each of the three functions. The vertical line in the image represents a particular temperature that the three arrows (truth values) gauge. Since the red arrow points to zero, this temperature may be interpreted as "not hot". The orange arrow (pointing at 0.2) may describe it as "slightly warm" and the blue arrow (pointing at 0.8) "fairly cold".

Linguistic variables

While variables in mathematics usually take numerical values, in fuzzy logic applications, the non-numeric *linguistic variables* are often used to facilitate the expression of rules and facts.[5]

A linguistic variable such as *age* may have a value such as *young* or its antonym *old*. However, the great utility of linguistic variables is that they can be modified via linguistic hedges applied to primary terms. The linguistic hedges can be associated with certain functions. For example, L. A. Zadeh proposed to take the square of the membership function. This model, however, does not work properly. For more details, see the references.

An example of fuzzy reasoning

Fuzzy Set Theory defines Fuzzy Operators on Fuzzy Sets. The problem in applying this is that the appropriate Fuzzy Operator may not be known. For this reason, Fuzzy logic usually uses IF-THEN rules, or constructs that are equivalent, such as fuzzy associative matrices.

Rules are usually expressed in the form:
IF *variable* IS *property* THEN *action*

For example, an extremely simple temperature regulator that uses a fan might look like this:

IF temperature IS very cold THEN stop fan
IF temperature IS cold THEN turn down fan
IF temperature IS normal THEN maintain level
IF temperature IS hot THEN speed up fan

Notice there is no "ELSE". All of the rules are evaluated, because the temperature might be "cold" and "normal" at the same time to different degrees.

The AND, OR, and NOT operators of boolean logic exist in fuzzy logic, usually defined as the minimum, maximum, and complement; when they are defined this way, they are called the *Zadeh operators*, because they were first defined as such in Zadeh's original papers. So for the fuzzy variables x and y:

NOT x = (1 - truth(x))
x AND y = minimum(truth(x), truth(y))
x OR y = maximum(truth(x), truth(y))

There are also other operators, more linguistic in nature, called *hedges* that can be applied. These are generally adverbs such as "very", or "somewhat", which modify the meaning of a set using a mathematical formula.

In application, the programming language Prolog is well geared to implementing fuzzy logic with its facilities to set up a database of "rules" which are queried to deduct logic. This sort of programming is known as logic programming.

Once fuzzy relations are defined, it is possible to develop fuzzy relational databases. The first fuzzy relational database, FRDB, appeared in Maria Zemankova's dissertation. Later, some other models arose like the Buckles-Petry model, the Prade-Testemale Model, the Umano-Fukami model or the GEFRED model by J.M. Medina, M.A. Vila et al. In the context of fuzzy databases, some fuzzy querying languages have been defined, highlighting the SQLf by P. Bosc et al. and the FSQL by J. Galindo et al. These languages define some structures in order to include fuzzy aspects in the SQL statements, like fuzzy conditions, fuzzy comparators, fuzzy constants, fuzzy constraints, fuzzy thresholds, linguistic labels and so on.

Other examples

- If a male is 1.8 meters, consider him as tall:

  IF male IS true AND height >= 1.8 THEN is_tall IS true; is_short IS false

- The fuzzy rules do not make sharp distinction between *tall* and *short*:

  IF height <= medium male THEN is_short IS agree somewhat
  IF height >= medium male THEN is_tall IS agree somewhat

In the fuzzy case, there are no such heights as 1.83 meters, but there are fuzzy values, like the following assignments:

dwarf male = [0, 1.3] m
short male = [1.3, 1.5] m
medium male = [1.5, 1.8] m
tall male = [1.8, 2.0] m
giant male > 2.0 m

For the consequent, there are may also be more than two values:

agree not = 0
agree little = 1
agree somewhat = 2
agree a lot = 3
agree fully = 4

In the binary (or "crisp") case, a person of 1.79 meters is considered of *medium* height, while another person who is 1.8 meters or 2.25 meters tall is considered *tall*.

The crisp example differs deliberately from the fuzzy one. The antecedent was not given fuzzy values:

IF male >= agree somewhat AND ...

as gender is often considered binary information.

Mathematical fuzzy logic

In mathematical logic, there are several formal systems of "fuzzy logic"; most of them belong among so-called t-norm fuzzy logics.

Propositional fuzzy logics

The most important propositional fuzzy logics are:

- Monoidal t-norm-based propositional fuzzy logic MTL is an axiomatization of logic where conjunction is defined by a left continuous t-norm, and implication is defined as the residuum of the t-norm. Its models correspond to MTL-algebras that are prelinear commutative bounded integral residuated lattices.
- Basic propositional fuzzy logic BL is an extension of MTL logic where conjunction is defined by a continuous t-norm, and implication is also defined as the residuum of the t-norm. Its models correspond to BL-algebras.

- Łukasiewicz fuzzy logic is the extension of basic fuzzy logic BL where standard conjunction is the Łukasiewicz t-norm. It has the axioms of basic fuzzy logic plus an axiom of double negation, and its models correspond to MV-algebras.
- Gödel fuzzy logic is the extension of basic fuzzy logic BL where conjunction is Gödel t-norm. It has the axioms of BL plus an axiom of idempotence of conjunction, and its models are called G-algebras.
- Product fuzzy logic is the extension of basic fuzzy logic BL where conjunction is product t-norm. It has the axioms of BL plus another axiom for cancellativity of conjunction, and its models are called product algebras.
- Fuzzy logic with evaluated syntax (sometimes also called Pavelka's logic), denoted by EVŁ, is a further generalization of mathematical fuzzy logic. While the above kinds of fuzzy logic have traditional syntax and many-valued semantics, in EVŁ is evaluated also syntax. This means that each formula has an evaluation. Axiomatization of EVŁ stems from Łukasziewicz fuzzy logic. A generalization of classical Gödel completeness theorem is provable in EVŁ.

Predicate fuzzy logics

These extend the above-mentioned fuzzy logics by adding universal and existential quantifiers in a manner similar to the way that predicate logic is created from propositional logic. The semantics of the universal resp. existential quantifier in t-norm fuzzy logics is the infimum resp. supremum of the truth degrees of the instances of the quantified subformula.

Higher-order fuzzy logics

These logics, called fuzzy type theories, extend predicate fuzzy logics to be able to quantify also predicates and higher order objects. A fuzzy type theory is a generalization of classical simple type theory introduced by B. Russell [6] and mathematically elaborated by A. Church [7] and L. Henkin[8] .

Decidability issues for fuzzy logic

The notions of a "decidable subset" and "recursively enumerable subset" are basic ones for classical mathematics and classical logic. Then, the question of a suitable extension of such concepts to fuzzy set theory arises. A first proposal in such a direction was made by E.S. Santos by the notions of *fuzzy Turing machine*, *Markov normal fuzzy algorithm* and *fuzzy program*. Successively, L. Biacino and G. Gerla showed that such a definition is not adequate and therefore proposed the following one. $Ü$ denotes the set of rational numbers in [0,1]. A fuzzy subset $s : S \rightarrow [0,1]$ of a set S is *recursively enumerable* if a recursive map $h : S\times N \rightarrow Ü$ exists such that, for every x in S, the function $h(x,n)$ is increasing with respect to n and $s(x) = \lim h(x,n)$. We say that s is *decidable* if both s and its complement $-s$ are recursively enumerable. An extension of such a theory to the general case of the L-subsets is proposed in Gerla 2006. The proposed definitions are well related with fuzzy logic. Indeed, the following theorem holds true (provided that the deduction apparatus of the fuzzy logic satisfies some obvious effectiveness property).

Theorem. Any axiomatizable fuzzy theory is recursively enumerable. In particular, the fuzzy set of logically true formulas is recursively enumerable in spite of the fact that the crisp set of valid formulas is not recursively enumerable, in general. Moreover, any axiomatizable and complete theory is decidable.

It is an open question to give supports for a *Church thesis* for fuzzy logic claiming that the proposed notion of recursive enumerability for fuzzy subsets is the adequate one. To this aim, further investigations on the notions of fuzzy grammar and fuzzy Turing machine should be necessary (see for example Wiedermann's paper). Another open question is to start from this notion to find an extension of Gödel’s theorems to fuzzy logic.

Application areas

- Automobile and other vehicle subsystems, such as automatic transmissions, ABS and cruise control (e.g. Tokyo monorail)
- Air conditioners
- The Massive engine used in the *Lord of the Rings* films, which helped huge scale armies create random, yet orderly movements
- Cameras
- Digital image processing, such as edge detection
- Rice cookers
- Dishwashers
- Elevators
- Washing machines and other home appliances
- Video game artificial intelligence
- Language filters on message boards and chat rooms for filtering out offensive text
- Pattern recognition in Remote Sensing
- Hydrometeor classification algorithms for polarimetric weather radar
- Fuzzy logic has also been incorporated into some microcontrollers and microprocessors, for instance, the Freescale 68HC12.

Mineral Deposit estimation

Controversies

Fuzzy logic is the same as "imprecise logic".

Fuzzy logic is not any less precise than any other form of logic: it is an organized and mathematical method of handling *inherently* imprecise concepts. The concept of "coldness" cannot be expressed in an equation, because although temperature is a quantity, "coldness" is not. However, people have an idea of what "cold" is, and agree that there is no sharp cutoff between "cold" and "not cold", where something is "cold" at N degrees but "not cold" at N+1 degrees — a concept classical logic cannot easily handle due to the principle of bivalence. The result has no set answer so it is believed to be a 'fuzzy' answer. Fuzzy logic simply provides a mathematical model of the vagueness which is manifested in the above example.

Fuzzy logic is a new way of expressing probability.

Fuzzy logic and probability are different ways of expressing uncertainty. While both fuzzy logic and probability theory can be used to represent subjective belief, fuzzy set theory uses the concept of fuzzy set membership (i.e. *how much* a variable is in a set), probability theory uses the concept of subjective probability (i.e. *how probable* do I think that a variable is in a set). While this distinction is mostly philosophical, the fuzzy-logic-derived possibility measure is inherently different from the probability measure, hence they are not *directly* equivalent. However, many statisticians are persuaded by the work of Bruno de Finetti that only one kind of mathematical uncertainty is needed and thus fuzzy logic is unnecessary. On the other hand, Bart Kosko argues that probability is a subtheory of fuzzy logic, as probability only handles one kind of uncertainty. He also claims to have proven a derivation of Bayes' theorem from the concept of fuzzy subsethood. Lotfi Zadeh argues that fuzzy logic is different in character from probability, and is not a replacement for it. He fuzzified probability to fuzzy probability and also generalized it to what is called possibility theory. Other approaches to uncertainty include Dempster-Shafer theory and rough sets.

Note, however, that fuzzy logic is not controversial to probability but rather complementary (cf. [9])

Fuzzy logic will be difficult to scale to larger problems.

This criticism is mainly because there exist problems with conditional possibility, the fuzzy set theory equivalent of conditional probability (see Halpern (2003), Section 3.8). This makes it difficult to perform inference. However there have not been many studies comparing fuzzy-based systems with probabilistic ones.

See also

- Artificial intelligence
- Artificial neural network
- Biologically-inspired computing
- Cloud computing
- Combs method
- Concept mining
- Contextualism
- Control system
- Defuzzification
- Dynamic logic
- Expert system
- FuzzyCLIPS expert system
- Fuzzy associative matrix
- Fuzzy concept
- Fuzzy Control System
- Fuzzy Control Language
- False dilemma
- Fuzzy electronics
- Fuzzy mathematics
- Fuzzy set
- Fuzzy subalgebra
- Machine learning
- → Multi-valued logic
- Neuro-fuzzy
- Paradox of the heap
- Perspectivism
- Pattern recognition
- Petr Hájek
- Rough set

- Type-2 fuzzy sets and systems
- Vagueness

Notes

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External links

Additional articles

- Formal fuzzy logic (http://en.citizendium.org/wiki/Formal_fuzzy_logic) - article at Citizendium
- Fuzzy Logic (http://www.scholarpedia.org/article/Fuzzy_Logic) - article at Scholarpedia
- Modeling With Words (http://www.scholarpedia.org/article/Modeling_with_words) - article at Scholarpedia
- Fuzzy logic (http://plato.stanford.edu/entries/logic-fuzzy/) - article at Stanford Encyclopedia of Philosophy
- Fuzzy Math (http://blog.peltarion.com/2006/10/25/fuzzy-math-part-1-the-theory) - Beginner level introduction to Fuzzy Logic.
- Fuzzy Logic and the Internet of Things: I-o-T (http://www.i-o-t.org/post/WEB_3)

Links pages

- Web page about FSQL (http://www.lcc.uma.es/~ppgg/FSQL/): References and links about FSQL

Software & tools

- DotFuzzy: Open Source Fuzzy Logic Library (http://www.havana7.com/dotfuzzy)
- JFuzzyLogic: Open Source Fuzzy Logic Package + FCL (sourceforge, java) (http://jfuzzylogic.sourceforge.net/)
- pyFuzzyLib: Open Source Library to write software with fuzzy logic (Python) (http://sourceforge.net/projects/pyfuzzylib)
- RockOn Fuzzy: Open Source Fuzzy Control And Simulation Tool (Java) (http://www.timtomtam.de/rockonfuzzy)
- Free Educational Software and Application Notes (http://www.fuzzytech.com)
- InrecoLAN FuzzyMath (http://www.openfuzzymath.org), Fuzzy logic add-in for OpenOffice.org Calc
- Open fuzzy logic based inference engine and data mining web service based on Metarule (http://www.metarule.com)
- Open Source Software "mbFuzzIT" (Java) (http://mbfuzzit.sourceforge.net)

Tutorials

- Fuzzy Logic Tutorial (http://www.jimbrule.com/fuzzytutorial.html)
- Another Fuzzy Logic Tutorial (http://www.calvin.edu/~pribeiro/othrlnks/Fuzzy/home.htm) with MATLAB/Simulink Tutorial
- Fuzzy logic in your game (http://www.byond.com/members/DreamMakers?command=view_post&post=37966) - tutorial aimed towards game programming.
- Simple test to check how well you understand it (http://www.answermath.com/fuzzymath.htm)

Applications

- Research article that describes how industrial foresight could be integrated into capital budgeting with intelligent agents and Fuzzy Logic (http://econpapers.repec.org/paper/amrwpaper/398.htm)
- A doctoral dissertation describing how Fuzzy Logic can be applied in profitability analysis of very large industrial investments (http://econpapers.repec.org/paper/pramprapa/4328.htm)

Research Centres

- Institute for Research and Applications of Fuzzy Modeling (http://irafm.osu.cz/)
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