# A List of Recommended Books in Topology

Allen Hatcher

These are books that I personally like for one reason or another, or at least find useful. They range from elementary to advanced, but don't cover absolutely all areas of Topology. The number of Topology books has been increasing rather rapidly in recent years after a long period when there was a real shortage, but there are still some areas that are difficult to learn due to the lack of a good book.

The list was made in 2003 and is in need of updating. For the books that were still in print in 2003 I gave the price at that time since this certainly seems like relevant information. One cannot help noticing the wide variation in prices, which shows how ridiculously inflated the prices from some publishers are. When books are available online for free downloading this has been indicated. Unfortunately the number of such books is still small.

Here are the main headings for the list:



## I. Introductory Books.

### General Introductions .

Here are two books that give an idea of what topology is about, aimed at a general audience, without much in the way of prerequisites.

- V V Prasolov. *Intuitive Topology.* American Mathematical Society 1995. [$20]
- J R Weeks. *The Shape of Space.* 2nd ed. Marcel Dekker, 2002. [$35]

**Point-Set Topology.**

The standard textbook here seems to be the one by Munkres, but I've never been able to work up any enthusiasm for this rather pedestrian treatment. Also it's now quite expensive at $98. Instead I prefer the following books:

- K Jänich. *Topology*. Springer, 1984. [$30]

  — A pleasure to read.

- M A Armstrong. *Basic Topology*. Springer, 1983. [$48]

- J Dugundji. *Topology*. Boston: Allyn and Bacon, 1966. [OP]

  — A fine reference book on point-set topology, now out of print, unfortunately.

- TW Gamelin and RE Greene. *Introduction to Topology*. 2nd ed. Dover Publications, 1999. [$11]

- O Viro, O Ivanov, V Kharlamov, and N Netsvetaev. *Elementary Topology*. http://www.math.uu.se/~oleg/educ-texts.html

  — Essentially just an outline with proofs left as exercises, but with many insightful comments. Includes also some algebraic topology and manifold theory.

**II. Algebraic Topology.**

**Introductory.**

Naturally my favorite here is:

- A Hatcher. *Algebraic Topology*. Cambridge University Press, 2002. [$30]
  Free electronic version available at http://www.math.cornell.edu/~hatcher

Here are three more with different viewpoints:

- J P May. *A Concise Course in Algebraic Topology*. University of Chicago Press, 1999. [$18]

  — Good for getting the big picture. Perhaps not as easy for a beginner as the preceding book.

- G E Bredon. *Topology and Geometry*. Springer GTM 139, 1993. [$70]

  — Includes basics on smooth manifolds, and even some point-set topology.

- R Bott and L W Tu. *Differential Forms in Algebraic Topology*. Springer GTM 82,

1982. [$60]

— Develops algebraic topology from the point of view of differential forms. Includes a very nice introduction to spectral sequences.

**Vector Bundles, Characteristic Classes, and K–Theory**

For these topics one can start with either of the following two books, the second being the classical place to begin:

- A Hatcher. *Vector Bundles and K–Theory.* Unfinished book available online at http://www.math.cornell.edu/~hatcher
- J W Milnor and J D Stasheff. *Characteristic Classes.* Annals of Math Studies 76. Princeton University Press, 1974. [$65]

For an introduction to K–theory the classical alternative to the first of the two preceding books is:

- M Atiyah. *K–Theory.* Perseus, 1989. [Originally published by W.A. Benjamin in 1967.] [$55]

**More Advanced Topics.**

Again listing my favorites first, we have:

- A Hatcher. *Spectral Sequences in Algebraic Topology.* Unfinished book available online at http://www.math.cornell.edu/~hatcher
- J F Adams. *Infinite Loop Spaces.* Annals of Math Studies 90. Princeton University Press, 1978. [$30]
- D C Ravenel. *Complex Cobordism and Stable Homotopy Groups of Spheres.* Academic Press, 1986. [OP, to be reprinted by AMS.]
- R E Mosher and M C Tangora. *Cohomology Operations and Applications in Homotopy Theory.* Harper and Row, 1968. [OP]
- S O Kochman. *Bordism, Stable Homotopy, and Adams Spectral Sequences.* Fields Institute Monographs 7. AMS, 1996. [$49]
- Y Rudyak. *On Thom Spectra, Orientability, and Cobordism.* Springer, 1998. [$139]
- R E Stong. *Notes on Cobordism Theory.* Princeton University Press, 1968. [OP]

  — An older book emphasizing the calculations of the coefficient rings of various forms of

cobordism.

- D C Ravenel. *Nilpotence and Periodicity in Stable Homotopy Theory.* Annals of Math Studies 128. Princeton University Press, 1992. [$43]
- J F Adams. *Stable Homotopy and Generalised Homology.* University of Chicago Press, 1974. [$34]
- F Hirzebruch, T Berger, and R Jung. *Manifolds and Modular Forms.* Vieweg, 1992. [OP?]

  — One of the few textbook sources for the slowly emerging and potentially very important topic of elliptic cohomology.
- P Hilton, G Mislin, and J Roitberg. *Localization of Nilpotent Groups and Spaces.* North-Holland, 1975. [OP]

  — The standard source for classical localization. The newer generalizations haven't yet filtered down to the textbook level.
- Y Félix, S Halperin, and J-C Thomas. *Rational Homotopy Theory.* Springer GTM 205, 2001. [$60]
- P A Griffiths and J W Morgan. *Rational Homotopy Theory and Differential Forms.* Birkhäuser, 1981. [OP]
- J P May. *Simplicial Objects in Algebraic Topology.* Van Nostrand, 1967. Reprinted by University of Chicago Press, 1982 and 1992. [$20]
- M Mimura and H Toda. *Topology of Lie Groups.* Translations of Mathematical Monographs 91. AMS, 1991. [$51]

  — Includes the algebraic topology proof of Bott Periodicity, as well as information about the five exceptional Lie groups.
- R M Kane. *The Homology of Hopf Spaces.* North-Holland, 1988. [$176]

  — Look at that price! And it's not even in Tex. But a nice book otherwise.
- JR Harper. *Secondary Cohomology Operations.* AMS, 2002. [$49]
- J McCleary. *A User's Guide to Spectral Sequences.* 2nd ed. Cambridge University Press, 2001. [$37]

  — A technical handbook, not as user-friendly as one might wish, and with some glaring errors, but included here in the absence of other sources.

- A Adem and R J Milgram. *Cohomology of Finite Groups.* Springer, 1994. [OP]
- D J Benson. *Representations and Cohomology, Volume II: Cohomology of Groups and Modules.* Cambridge University Press, 1992. [$35]
- W G Dwyer and H-W Henn. *Homotopy Theoretic Methods in Group Cohomology.* Birkhäuser, 2001. [$30]

  — Two separate sets of notes for short courses by the two authors, each about 50 pages.

**III. Manifold Theory.**

**Differential Topology.**

For expositional clarity Milnor's three little books can hardly be beaten:

- J Milnor. *Topology from the Differentiable Viewpoint.* rev. ed. Princeton University Press, 1997. [$15]

  — Quite elementary and accessible. Just 65 pages, so only a small amount of material is covered, alas.
- J Milnor. *Morse Theory.* Annals of Math Studies 51. Princeton University Press, 1963. [$50]
- J Milnor. *Lectures on the h-Cobordism Theorem.* Princeton University Press, 1965. [OP]

  — A more specialized topic, but a cornerstone of the subject.

An alternative to Milnor's Morse Theory book that goes farther is:

- Y Matsumoto. *An Introduction to Morse Theory.* Translations of Mathematical Monographs 208. AMS, 2002. [$39]

At a somewhat more advanced level there is:

- A A Kosinski. *Differential Manifolds.* Academic Press, 1993. [$98]. Dover reprint, 2007. [$16]

  — A rather nice exposition that for some reason has never really become popular. Perhaps the price had something to do with this. Fortunately there is now an inexpensive Dover reprint.

**Piecewise Linear Topology.**

PL topology was popular in the early days of manifold theory, but with the development of the appropriate tools in the purely topological category the PL category has fallen out of favor. The best source for this classical subject seems to be:

- C P Rourke and B J Sanderson. *Introduction to Piecewise-Linear Topology*. Springer, 1972. [OP]

**Topological Manifolds.**

A textbook exposition is still lacking here, probably because of the technical difficulty of the subject. Here are an early monograph and a recent survey article:

- R C Kirby and L C Siebenmann. *Foundational Essays on Topological Manifolds, Smoothings, and Triangulations.* Annals of Math Studies 88. Princeton University Press, 1977. [$45]
- Y Rudyak. *Piecewise Linear Structures on Topological Manifolds.* preprint available at arXiv:math. AT/0105047, 2001.

**Surgery Theory.**

Surgery theory addresses the basic problem of classifying manifolds up to homeomorphism or diffeomorphism. The first 100 pages of the following book give a nice overview:

- S Weinberger. *The Topological Classification of Stratified Spaces.* University of Chicago Press, 1994. [$20]

A more systematic exposition can be found in:

- A Ranicki. *Algebraic and Geometric Surgery*. Oxford University Press, 2002. [$110]

A collection of surveys that may be helpful is:

- S Cappell, A Ranicki, and J Rosenberg, eds. *Surveys on Surgery Theory.* Annals of Math Studies 145. Princeton University Press, 2000. [$45]

And here are two of the original books:

- W Browder. *Surgery on Simply-Connected Manifolds.* Springer, 1972. [OP]
- C T C Wall. *Surgery on Compact Manifolds, 2nd ed.* AMS, 1999. [$59]

— This new edition of the 1970 original adds some commentary by A Ranicki.

One of the classic early papers could also serve rather well to give the flavor of surgery theory:

- M A Kervaire and J W Milnor. *Groups of homotopy spheres.* Annals of Mathematics 77 (1963), 504-537.

**IV. Low-Dimensional Topology.**

**Surfaces.**

Someone should someday write a comprehensive exposition of topological surface theory. A small fraction of the theory can be found in

- A J Casson and S A Bleiler. *Automorphisms of Surfaces after Nielsen and Thurston.* LMS Student Texts 9. Cambridge University Press, 1988. [$15]

One can also look at an original paper:

- W Thurston. *On the geometry and dynamics of diffeomorphisms of surfaces.* Bull. Amer. Math. Soc. 19 (1988), 417-431.

**3-Manifolds.**

For 3-manifold theory there are several books:

- W Thurston. *Three-Dimensional Geometry and Topology.* Princeton University Press, 1997. [$55]

  — A geometric introduction by the master. Also useful for the geometry of surfaces.

- A Hatcher. *Basic Topology of 3-Manifolds.* Unpublished notes available online at http://www.math.cornell.edu/~hatcher

  — The more classical topological aspects of 3-manifold theory.

- J Hempel. *3-Manifolds.* Annals of Math Studies 86. Princeton University Press, 1976. [$30]

- P Scott. *Geometries of 3-manifolds.* Bull. London Math. Soc. 15: 401-487, 1983.

  — A clear presentation of seven of Thurston's eight possible geometric structures on 3-manifolds, all but hyperbolic geometry, the most subtle case by far.

- N Saveliev. *Invariants for Homology 3-Spheres.* Springer, 2002. [$99]

- M Kapovich. *Hyperbolic Manifolds and Discrete Groups.* Birkhäuser, 2001. [$77]

**Knot Theory.**

This is a very accessible topic and there are plenty of readable books. The following ones are listed more or less in order of increasing sophistication.

- C C Adams. *The Knot Book.* W.H. Freeman, 1994. [$17]
- C Livingston. *Knot Theory.* Carus Mathematical Monographs 24. Mathematical Association of America, 1993. [$40]
- J Roberts. *Knots Knotes.* http://math.ucsd.edu/~justin/papers/knotes.pdf
- D Rolfsen. *Knots and Links.* Publish or Perish, 1976. Recently reprinted by the AMS.

  — For a long time the standard textbook, noteworthy for its readability and nice pictures. Still useful for the more classical theory.
- W B R Lickorish. *An Introduction to Knot Theory.* Springer GTM 175, 1997. [$53]
- V V Prasolov and A B Sossinsky. *Knots, Links, Braids and 3-Manifolds.* Translations of Mathematical Monographs 154. AMS, 1997. [$51]

  — Covers also some general 3-manifold theory relevant to knot theory. Emphasizes ideas and intuition.
- G Burde and H Zieschang. *Knots.* 2nd ed. de Gruyter, 2003. [$98]
- A Kawauchi. *A Survey of Knot Theory.* Birkhäuser, 1996. [$99]

**4-Manifolds.**

Topology is only part of the story here. Two books that focus on this part are:

- M H Freedman and F Quinn. *Topology of 4-Manifolds.* Princeton University Press, 1990. [OP]
- R E Gompf and A I Stipsicz. *4-Manifolds and Kirby Calculus.* AMS, 1999. [$65]

**V. Miscelllaneous.**

- J Matoušek. *Using the Borsuk-Ulam Theorem.* Springer, 2003 [$50]

  — A whole book on this classic theorem and its many varied applications in geometry and combinatorics. Highly readable and highly recommended.

- J Dieudonné. *A History of Algebraic and Differential Topology 1900-1960.* Birkhäuser, 1989.

  — More than just a history, it also explains the major ideas as they were first developed.

Here are three super-expensive compilations of articles by many authors:

- I M James, ed. *History of Topology.* North-Holland, 1999. [$178]
- I M James, ed. *Handbook of Algebraic Topology.* North-Holland, 1995. [$222]
- R J Daverman and R B Sher, eds. *Handbook of Geometric Topology.* North-Holland, 2002. [$175]

Finally, here are four classics that are included in this list mainly for their historical interest:

- S Eilenberg and N Steenrod. *Foundations of Algebraic Topology.* Princeton University Press, 1952.

  — The first book to give the axiomatic viewpoint toward homology and cohomology theory.

- E H Spanier. *Algebraic Topology.* McGraw-Hill, 1966. Reprinted by Springer. [$50]

  — The standard reference up until the past decade or so when other more accessible and up to date books began to appear.

- P J Hilton and S Wylie. *Homology Theory.* Cambridge University Press, 1967.

  — One of the standard expositions of homology and cohomology from its time. Contains some interesting unsuccessful attempts at reforming terminology and notation.

- N Steenrod. *The Topology of Fiber Bundles.* Princeton University Press, 1951. [$28]

  — The original exposition of this basic topic. Still quite readable.