PENNSTATE 1855

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Facts About the Octacube Sculpture

written by Adrian Ocneanu, October 2005

Click on image below to view animation of a hypercube rotating in four dimensions, in windowed radial stereographic projection.

Click on image below to view animation.

Animation, Octacube design and 4D projection method © A.Ocneanu

For pictures of the Open House and dedication, click here.

Open House and Dedication of Unique Sculpture to be Hosted by Department of Mathematics on 21 October

Click here to watch the IN-MOTION video production, "Portal to the Fourth Dimension."

13 October 2005—The Penn State Department of Mathematics will host an open house of its extensively renovated McAllister Building, featuring a dedication ceremony for a unique sculpture with deep mathematical significance on 21 October 2005 at the Penn State University Park campus. The event will begin at 3:30 p.m. with a ceremony to dedicate the "Octacube" sculpture in the first-floor atrium of McAllister Building, followed by an opportunity for participants to explore the renovated building until 5:00 p.m. No good rendering of any 4-dimensional object existed anywhere in the world before the Octacube, either in solid or virtual form, according to **Adrian Ocneanu**, the Penn State professor of mathematics who designed the sculpture.

In addition to the events on 21 October, the mathematics department will host a mathematical talk on 20 October at 4:00 p.m. and a talk for the general public on 26 October at 6:00 p.m. All three events will take place in the atrium near the sculpture, will feature 4-dimensional movies, and will be open to the public at no charge.

Click on image for larger view.

The sculpture measures about six feet in every direction and is mounted on a granite base about three feet high. Click here for more information.

The sculpture is a gift from **Jill Grashof Anderson**, a mathematics alumna of Penn State as a memorial for her husband, **Kermit C. Anderson**, also a Penn State mathematics graduate, who was killed in the terrorist attack on the World Trade Center in New York City on 11 September 2001. The dedication ceremony for the sculpture will include an explanation of its mathematical meaning by its designer, **Adrian Ocneanu**, professor of mathematics. Ocneanu is a mathematics researcher who works on models of Quantum Field Theory based on symmetry. He designed the sculpture as a research and teaching tool related to his research. The stainless-steel Octacube is a striking object of visual art and also a mental portal to the fourth dimension, a teaching tool, and a research object bringing together many branches of mathematics and physics connected to the structure of symmetry.

The sculpture, which measures about six feet in every direction, presents the three-dimensional "shadow" of a four-dimensional solid object. "Although mathematicians can work with a fourth dimension abstractly by adding a fourth coordinate to the three that we use to describe a point in space, a fourth spatial dimension is difficult to visualize," Ocneanu explains. "The sculpture was designed with a new method which captures four dimensional symmetry better than anything done before." The Octacube was produced by the staff of the Engineering Services Shop, managed by **Jerry Anderson**. "It is rare that we get a chance to produce something so extraordinary for people to enjoy," Jerry Anderson says. "The Octacube demonstrates the high level of skill and craftsmanship of the Penn State people who transformed it from a design to an object, including **Janet Page**, **James Kustenborder**, **Ronald Weaver**, **Brian Bennett**, **Dennis Praskovich**, **Thomas Coakley**, **Thomas Rimmey** and **Lee Brooks**." Jill Grashof Anderson says she hopes the sculpture will encourage students, faculty, administrators, alumnae, and friends to ponder and appreciate the world of mathematics. "I also hope that all who view the sculpture will begin to grasp the sobering fact that everyone is vulnerable to something terrible happening to them and that we all must learn to live one day at a time, making the very best of what has been given to us." She adds, "It would be great if everyone who views the Octacube walks away with the feeling that being kind to others is a good way to live."

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MORE INFORMATION
More information about the Octacube sculpture is available below.

CONTACTS:
Jill Grashof Anderson can be reached via Suzanne Grieb at sds6@psu.edu or 814-863-4683.
Adrian Ocneanu: 814-865-4061, adrian@math.psu.edu
Jerry Anderson: 814-865-4963, janderson@engr.psu.edu

New Sculpture Looks Beyond Three Dimensions

Click on image for larger view

13 October 2005—Artistic works traditionally carry significance beyond their physical beauty, but a new sculpture in the McAllister Building headquarters of the Penn State Department of Mathematics may carry that tradition to its limits. The stainless-steel work, a striking object of visual art, also is a mental portal to the fourth dimension, a teaching tool, a memorial to a graduate of the math department, and a reminder of the terrorist attacks of 11 September 2001. The sculpture itself measures about six feet in every direction and is mounted on a granite base about three feet high in order to bring its center approximately to eye level.

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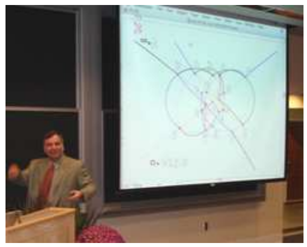
Click here for more information.

The sculpture, designed by **Adrian Ocneanu**, professor of mathematics at Penn State, presents a three-dimensional "shadow" of a four-dimensional solid object. Ocneanu's research involves mathematical models for quantum field theory based on symmetry. One aspect of his work is modeling regular solids, both mathematically and physically. In the three-dimensional world, there are five regular solids--tetrahedron, cube, octahedron, dodecahedron, and icosahedron--whose faces are composed of triangles, squares, or pentagons. In four dimensions, there are six regular solids, which can be built based on the symmetries of the three-dimensional solids. Unfortunately, humans cannot process information in four dimensions directly because we don't see the universe that way. Although mathematicians can work with a fourth dimension abstractly by adding a fourth coordinate to the three that we use to describe a point in space, a fourth spatial dimension is difficult to visualize. For that, we need models. "Four-dimensional models are useful for thinking about and finding new relationships and phenomena," says Ocneanu. "The process is actually quite simple--think in one dimension less." To explain this concept, he points to a map. While the Earth is a three-dimensional object, its surface can be represented on a flat two-dimensional map.

Click on image for larger view.



Dr. Ocneanu, pictured at the inauguration of the Octacube, sharing his first apparition of the octacube as a wire model.

Ocneanu's sculpture similarly maps the four-dimensional solid into a space perceptible to the human observer. His process, radial stereography, presents a new way of making this projection. He explains the process by analogy to mapping a globe of the Earth onto a flat surface. "We place a light bulb at the north pole of the Earth and we project onto a sheet of paper placed underneath it," he says. "The southern hemisphere, away from the north pole, will remain quite small, while the northern hemisphere, near the projection pole, will become very big and north pole itself will be sent toward infinity." The technique can be used to make a two-dimensional projection of a cube by first mapping the cube radially onto the surface of a globe. Ocneanu explains, "The edges of our cube become circles on the map, just like straight highways are slightly curved on maps of the Earth. Its angles, however, remain true in this projection, so the map retains the key aspects of the symmetry of the original cube, unlike a photograph of a cube."

When the same technique is applied to project a four-dimensional solid into three dimensions, the inner part of the projection--equivalent to the south pole on the map--has smaller, undistorted faces, while the outer part extends toward infinity. Linear edges of the solid become circles in the projection. However, the projection is conformal, which means that the angles between faces and the way that the faces meet at corners are uniform throughout the projection. The retention of these key characteristics makes the sculpture a powerful teaching tool in addition to a powerful aesthetic object. "When I saw the actual sculpture, I had quite a shock," says Ocneanu. "I never imagined the play of light on the surfaces. There are subtle optical effects that you can feel but can't quite put your finger on." The sculpture has significance in several areas of mathematics related to the study of symmetry, and it can represent structures that are fundamental to many branches of mathematics and physics.

"The sculpture is a new way to represent a classical mathematical object," says **Nigel Higson**, head of the Penn State Department of Mathematics. "For professionals the sculpture is very rich in meaning, but it also has an aesthetic appeal that anyone can appreciate. In addition, it helps to start conversations about abstract mathematical concepts--something that is generally hard to do with anyone other than another expert."

Click on image for larger view.

The subject of the projection is a regular 4-dimensional solid of intermediate complexity, which Ocneanu calls an "octacube." It has 24 vertices, 96 edges, and 96 triangular faces, which enclose 24 three-dimensional "rooms." Windows cut in faces allow the viewer to see within the structure, the same way that a window in a cubic room opens to the inside of the cube. Physically, the sculpture is a giant puzzle of 96 triangular pieces cut from stainless steel and bent into spherical shape. Ocneanu attributes the success of the project to the machinists and welders of Penn State's Engineering Services Shop, managed by **Jerry Anderson**. "It turned out much better than I could have imagined," Ocneanu says. "It's very hard to make 12 steel sheets meet perfectly--and conformally--at each of the 23 vertices, with no trace of welding left. The people who built it are really world-class experts and perfectionists--artists in steel."

The sculpture was sponsored by Jill Grashof Anderson, a 1965 graduate of the mathematics department, who provided funds for its development and construction. It is dedicated to the memory of her husband, **Kermit Anderson**--also a 1965 mathematics graduate--who was killed in the World Trade Center terrorist attack on 11 September 2001. She also has sponsored a scholarship in his memory. "I hope that the sculpture will encourage students, faculty, administrators, alumnae, and friends to ponder and appreciate the wonderful world of mathematics," says Anderson. "I also hope that all who view the sculpture will begin to grasp the sobering fact that everyone is vulnerable to something terrible happening to them and that we all must learn to live one day at a time, making the very best of what has been given to us. It would be great if everyone who views the Octacube walks away with the feeling that being kind to others is a good way to live."

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Location

The sculpture is on public display in the lobby of the Mathematics Department in the McAllister building, between the HUB and the Old Main in the central part of the Pennsylvania State University campus.

Mapping
The sculpture represents a three dimensional map of the surface of a four dimensional regular solid. Adrian Ocneanu developed and copyrighted the map, called windowed radial stereographic projection. The projection is the first good method for representing four dimensional solids as it shows the 2d walls of the 3d rooms, not only their 1d scaffolding. The edge and corner angles between walls and are all equal, preserving the 4d symmetry of the model.

The Earth is 3-dimensional and we can make a map of its surface on a 2 dimensional sheet of paper. In a similar way in one dimension higher, the sculpture represents a map in our 3-dimensional space of the surface of a 4-dimensional body. On a map of the Earth the countries are 2d polygons, in the sculpture they are 3d rooms, so the sculpture is similar to an apartment building. Windows are cut into the walls before the projection, making the inner rooms of the sculpture visible.

On a map of the 3d Earth the angles between roads are maintained, while the scale of the map varies in order to stretch the surface of the sphere on a sheet - a map with these properties is called conformal in mathematics. A good conformal map is the stereographic projection, in which a light bulb is placed at the north pole and the shadow of the globe surface is taken on a sheet underneath the south pole. The southern hemisphere remains small in the middle of the map while the northern hemisphere, close to the projection pole, becomes big and surrounds the center of the map. If the initial object is not spherical, it is first radially projected on the surface of a sphere surrounding it, with a light bulb placed in the center of the sphere.

The surface of the 4-dimensional solid represented by the sculpture is first projected radially on a sphere and then stereographically in our 3d world which plays the role of a flat map. The stereographic projection is conformal in any dimension, and explains why the angles between walls and at the corners are constant throughout the sculpture. The inner part of the sculpture, image of the southern hemisphere, is small while the outer part imaging the northern hemisphere near the projection pole is large. A vertex of the octacube is actually located on the projection pole, so the walls surrounding it are cut half way to the pole and their large shadows give the outer legs of the sculpture extending toward infinity.

As a consequence of the symmetry preserved in the sculpture, the reflection in the flat faces of the front of the sculpture matches the parts behind, thus giving the stainless steel the feeling of transparency.

Name
The sculpture is titled "Octacube" as its 24 vertices consist of the 16 vertices of the 4 dimensional cube, also known as hypercube or tesseract, together with the centers of its 8 rooms, which form the vertices of the 4d octahedron.

Construction
Adrian Ocneanu wrote the software which gave the cutting instructions for 1/8" stainless steel sheets to the machines in the Penn State Engineering Shop. The machinists then worked for almost a year to bend into spherical shapes and weld the 96 triangular pieces which meet in a giant puzzle, 12 at each of the 23 vertices of the sculpture - the 24th vertex is at infinity. The sculpture measures 6 x 6 x 6 ft and weighs 1200 lbs.

Financing
The sculpture is a gift from Jill Anderson in memory of her husband Kermit killed in the terrorist attacks of 9/11/2001, "Lest we forget."

Mathematical structures
The sculpture was chosen because it encodes more mathematical structures in different branches of mathematics and physics than any other 4 dimensional object. It starts as a map of the 3d surface of the 4th of the 6 four dimensional regular solids, called the octacube, which has 24 vertices, 96 edges, 96 triangular faces and 24 octahedral 3d rooms.

As the angles between any two walls are 120º, and the walls are spherical or flat, the rooms of the sculpture could be in principle realized as soap bubbles, and are described mathematically as minimal surfaces.

The vertices of the sculpture are the centers of the 24 spheres which can surround a sphere in 4 dimensions, in a very tight pattern possible only in dimensions 2,4, 8 and 24. Five of these spheres intersect the sculpture base on circles which are engraved in the granite surface of the base. Sphere packings are important in number theory and cryptography.

The nodes and mid 3d rooms of the sculpture encode the rotational symmetries of a usual cube. Each point V encodes a rotation of angle alpha around the axis OV with the length OV equal to the tangent of alpha/4. The vertices vertices of a cube are divided according to parity into 2 tetrahedra. Rotations of the cube which send each of these tetrahedra into itself encode the vertices of the octacube. Rotations of the cube which switch the two tetrahedra encode the mid rooms of the octacube.

The vertices of the sculpture encode also a crystallographic structure called a root system of type D4 which describes how to build a higher dimensional symmetry group from 24 two dimensional components located at the vertices of the sculpture, in a branch of mathematics called Lie groups and algebras. Vertices are arranged 6 on each of the 12 circles and 4 lines of the sculpture, with each segment of a circle or line representing an angle of 60º of a circle in 4d, and with vertices not on the same circle being perpendicular to each other. Using in addition to the 24 vertices of the sculpture also part or all of the 24 mid room points, one obtains the same way the crystallographic root systems of type B4, C4 and F4.

Finally the symmetry structure called inversion in the unit sphere between the inner half and the outer half of the sculpture can be used to explain the relation between spin 1/2 and spin 1 structures in mathematical physics.

From left to right: Adrian Ocneanu, Jill Anderson, and Nigel Higson.

Pictured above: One of the original cast iron columns, circa 1904, restored during the 2005 renovations to the McAllister Building. The column was given to the Department of Mathematics by Alexander Building Construction, LLC.

Over 100 people turned out to admire the Octacube sculpture and the newly renovated McAllister Building.